CONTAINER LOADING PROBLEM MODEL DEVELOPMENT CONSIDERING WEIGHT DISTRIBUTION

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ABSTRACT

One of the biggest problems in logistic operation is material handling. By having good material handling, product demanded by costumer can reach the costumer's hand without any or with less damage. Container is one of material handling tools designed to move materials in one big unit load, but the container stability in weight is a problem that has to be considered for the safeties and easiness when the container will be transported to another place or location. This paper proposes an analytical model to optimize the weight distribution in one container. The model has capability in optimizing the cargo position inside the container to get the most stable cargo weight distribution. A case study example is also presented to illustrate the model.

Keywords: container loading problem; cargo weight distribution; center of mass; analytical model.

INTRODUCTION

Logistic is an important activity in industries. The existence of logistic in supply chain context is to move and put the inventory to reach specified time, place and profit objective in order to minimize total cost. Furthermore, in order to develop it, industries should have to integrate logistic activities, such as order processing, inventory, warehousing, material handling, packing, and transportation, to get a maximum benefit (Bowersox et al., 2007).

Moreover, logistic is not only an activity to deliver goods from one place to the other, but also to maintain goods' quality. Therefore, there are, at least, three consideration aspects in logistic problem, material handling, packing and delivering goods. Optimization on material handling and packing will affect to the improvement of logistic system and the achievement of the goals (Saputra, 2008).

Container is a tool of materials handling to move materials in one big unit load. Moreover, container can be adjusting to various transportation methods. Containers are increasingly set to replace traditional stowage in holds. Containerization of cargoes is becoming ever more widespread worldwide and almost all products are now transported by container (http://www.tis-gdv.de/tis_e/containe/inhalt1.htm Retrieved 28/03/08 World Wide Web on Saputra, 2008).

Optimization on material handling and packing using containers can be solved by container loading problem (CLP) approach. CLP focused on handling and packing square-shape boxes to fit in to one or more containers, such the container volume is optimally utilized (Pisinger, 2002). The main purpose of CLP is to minimize the number of containers used. However, there are a lot of factors to be considered as an approach to solve the problem. Bischoff dan Ratcliff (1995) explains there are, at least, 12 approaches have been used in order to solve CLP. One of them is weight distribution.

Weight distribution approach considers the center point mass of the cargos lies close to the geometrical midpoint of the container floor. According to the rules that apply in transportation in general, cargo weight must be distributed evenly over the inside of a container and must not exceed the maximum weight of cargo that has been set for a container used (http://www.geestline.com/quotation_clauses, retrieved 31/ 03/08 World Wide Web). The container will be moved by a material handling device. In addition, container is usually stored in a way stacked during stay in port or when transported by ship or plane. If the weights of the cargos in the container are not uniformly distributed, then the container will not be balanced and may cause the container tumbling over. This problem can affect the quality of goods carried in containers.

This paper's principal aim is to develop a model about handling and packing material problem in Indonesia's Aircraft manufacturing Company, PT. Dirgantara Indonesia (PT. DI). As a subcontractor for Airbus, PT. DI has a contract to make wing components for Airbus A380 until 2010. All A380's wing components are packed in 34 types of boxes and are sent by plane, every two weeks. This paper uses the results from research performed by Kustiawan (2007). In his research, Kustiawan (2007) stated that all boxes can be loaded in one container, L5G1 type, only. Thus, this paper proposed a model that can give an arrangement of boxes in container L5G1 type that has better cargo weight distribution than the arrangement proposed by Kustiawan in the same container.

The equilibrium of the cargos in the container will be obtained with the equitable weight distribution of cargos, which will be achieved through center point of mass approach. The model proposes the cargos in the container center point of mass coincide or at least close to the container center point of mass. Center point of mass can be interpreted as an equilibrium point of an object, which the moment due to the gravitational force on the object is equilibrium, the force resultant equal to zero (http://www.space-electronics.com/KnowHow/Glossary_Mass_Prop.php , retrieved 18/08/08 World Wide Web). On its equilibrium, the container will be stable, so it can maintain the quality of the cargos.

EQUILIBRIUM CONCEPT AND CENTER POINT OF MASS

Equilibrium Concept

A set of objects is on its equilibrium when there are no forces that can make the objects move, both translational motion or rotational (http://hyperphysics.phy-astr.gsu .edu / hbase / torq.html # equi, Retrieved 15/08/08 World Wide Web). To achieve equilibrium, the following conditions must be met:

1. The force resultant is zero (0).
   In three dimensional force system there are three force components that works on an object, F_x, F_y, and F_z. The visualization of the forces in three dimensional object is shown on the figure 1 below.

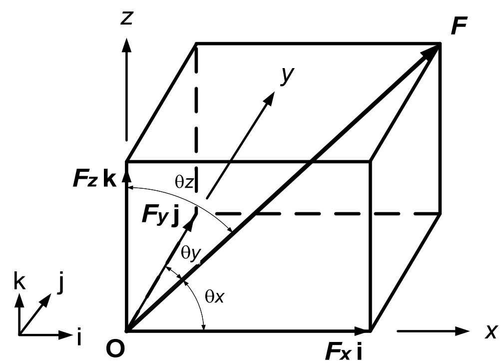


Figure 1 Visualization of forces at point O in three dimensional object (source: Meriam and Kraige (1987))

In order to reach equilibrium then the forces resultant on all axes must be zero (0).

$$\sum_i F_{x_i} = 0 \quad (1)$$

$$\sum_i F_{y_i} = 0 \quad (2)$$

$$\sum_i F_{z_i} = 0 \quad (3)$$

2. The torque resultant is zero (0).
   Torque is a vector that measures the tendency of a force to rotate the object on an axis (Serway et. Al. On http://en.wikipedia.org/wiki/Torque, Retrieved 15/08/08 World Wide Web). The value of the torque on an object is defined as the result of multiplication of forces by the distance between the objects center point of mass with the origin of the forces that worked on these objects. The visualization of the torque in three dimensional object is shown on the figure 2 below.

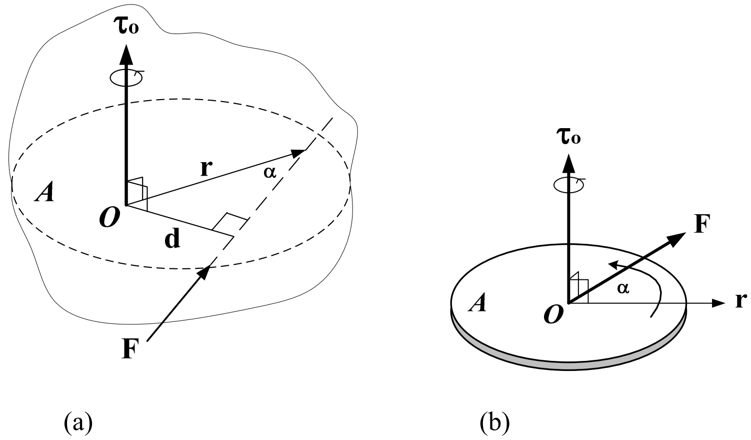


Figure 2 Visualization of torque τ_O at point O in three dimensional object (source: Meriam and Kraige (1987))

In order to reach equilibrium then the torque resultant on all axes must be zero (0).

$$\sum_i \tau_{x_i} = 0 \quad (4)$$

$$\sum_i \tau_{y_i} = 0 \quad (5)$$

$$\sum_i \tau_{z_i} = 0 \quad (6)$$

Equation (1) ensures that the amount of the forces that push the objects to the left (x^- axis direction) is equal to the amount of forces that push the objects to the right (x^+ axis direction). Equation (2) ensures that the amount of the forces that push the objects forward (y^- axis direction) is equal to the amount of forces that push the objects backward (y^+ axis direction). While equation (3) ensures that the amount of the forces that push the objects down (z^+ axis direction) is equal to the amount of forces that push the objects upwards (z^+ axis direction). Equation (4), (5), and (6) ensures that the torques that work on the objects (either clockwise or reverse) are eliminating each other, in other words, the resultant of the torques is zero (0).

If a set of objects supported on its center point of mass, the forces and torques resultant is zero, so the set of objects is steady and equilibrium.

Center Point of Mass

Each object can be considered as a system consisting of a group of points, each of the point has a mass. Center point of mass of the object is a specific point where the mass of the object is concentrated or in other words, it is an equilibrium point of the object. The center point of mass can be determined by the following formula (http://ocw.gunadarma.ac.id/course/diploma-three-program/study-program-of-computer-engineering-d3/fisika-dasar-1/kesetimbangan, Retrieve 14/08/08 World Wide Web).

$$\lim_{\Delta m \to 0} \sum_{i=1}^{\infty} \Delta m_i = \int dm \tag{7}$$

In three dimensional object, the coordinates of the center of mass of an object is (x_{pm}, y_{pm}, z_{pm}), where:

$$x_{pm} = \frac{\int x \cdot dm}{m}, \tag{8}$$
perpendicular distance on x axis from the area of yz to the object center point of mass object

$$y_{pm} = \frac{\int y \cdot dm}{m}, \tag{9}$$
perpendicular distance on y axis from the area of xz to the object center point of mass object

$$z_{pm} = \frac{\int z \cdot dm}{m}, \tag{10}$$
perpendicular distance on z axis from the area of xy to the object center point of mass object

CLP MODEL

The model is the development of the analytical model developed by Kocjan and Holmström (2003). This paper develops the model on the objective function. Moreover, the application of this model to PT. DI problem will need to develop some of the constraints.

Problem

A modeling problem in this paper is to find an arrangement of cargos in single container considering weight distribution through center point of mass approach.

Constants and Parameters

Constant-constant that is used in the development of basic CLP model is as follows:

N	Total number of packed boxes.
M	An arbitrary big number or big M

Define the following parameters:

$(p_i, q_i, r_i,)$ Parameters indicating length, width and height of box *i*.
(L_j, W_j, H_j) Parameters indicating length, width and height of container *j*.
m_i Parameters indicating mass of box *i*.

Decision Variables

To create a mathematical model, the following variables need to be optimized:

(x_i, y_i, z_i) Continuous variables denoting front-left bottom corner of box *i*.

v_{ik} set of binary variables $(a_{ik}, b_{ik}, c_{ik}, d_{ik}, e_{ik}, f_{ik})$ to indicate the placement of boxes relative to each other. A value of one for the variable a_{ik} states that the box *i* is placed to the right of box *k*. The next five variables define if a box *i* is on the right of, behind, in front of, below or above the box *k*, respectively. The set of these six variables are defined only when $i < k$.

Analytic Model

In terms of the above defined variables the problem is formulated as the following non-integer linear programming problem.

$$\textit{minimize} \left(\left| \frac{\sum_{i=1}^{N} m_i \cdot (x_i + \frac{p_i}{2})}{\sum_{i=1}^{N} m_i} - \frac{L}{2} \right| + \left| \frac{\sum_{i=1}^{N} m_i \cdot (y_i + \frac{q_i}{2})}{\sum_{i=1}^{N} m_i} - \frac{W}{2} \right| + \left(\frac{\sum_{i=1}^{N} m_i \cdot (z_i + \frac{t_i}{2})}{\sum_{i=1}^{N} m_i} \right) \right) \quad (11)$$

Subject to

$$x_i + p_i \le x_k + (1 - a_{ik}) \cdot M \quad \text{for } k = 1,\dots, N,\ i = 1,\dots, k\text{-}1 \quad (12)$$

$$x_k + p_k \le x_i + (1 - b_{ik}) \cdot M \quad \text{for } k = 1,\dots, N,\ i = 1,\dots, k\text{-}1 \quad (13)$$

$$y_i + q_i \le y_k + (1 - c_{ik}) \cdot M \quad \text{for } k = 1,\dots, N,\ i = 1,\dots, k\text{-}1 \quad (14)$$

$$y_k + q_k \le y_i + (1 - d_{ik}) \cdot M \quad \text{for } k = 1,\dots, N,\ i = 1,\dots, k\text{-}1 \quad (15)$$

$$z_i + r_i \le z_k + (1 - e_{ik}) \cdot M \quad \text{for } k = 1,\dots, N,\ i = 1,\dots, k\text{-}1 \quad (16)$$

$$z_k + r_k \le z_i + (1 - f_{ik}) \cdot M \quad \text{for } k = 1,\dots, N,\ i = 1,\dots, k\text{-}1 \quad (17)$$

$$a_{ik} + b_{ik} + c_{ik} + d_{ik} + e_{ik} + f_{ik} \ge 1 \quad \text{for } j = 1,\dots, m,\ k = 1,..,N,\ i = 1,\dots, k\text{-}1 \quad (18)$$

$$x_i + p_i \le L_j \quad \text{for } i = 1,\dots, N,\ j = 1,\dots, m \quad (19)$$

$$y_i + q_i \le W_j \quad \text{for } i = 1,\dots, N,\ j = 1,\dots, m \quad (20)$$

$$z_i + r_i \le H_j \quad \text{for } i = 1,\dots, N,\ j = 1,\dots, m \quad (21)$$

$$a_{ik}, b_{ik}, c_{ik}, d_{ik}, e_{ik}, f_{ik} \in \{0,1\} \quad \text{for } k = 1,\dots, N,\ i = 1,\dots, k\text{-}1 \quad (22)$$

$$x_i, y_i, z_i \ge 0 \quad \text{for } i = 1,\dots, N \quad (23)$$

The objective function of the model is to minimize the gap of the coordinates of the cargos' and container's center point of mass. The constraints (12) – (17) ensure that no two boxes placed in the container overlap with each other. Moreover, constraint (18) states that two boxes in the container must be placed in at least one of the listed relations. Finally, constraints (19) – (21) guarantee that boxes are placed within the physical size of the container (Kocjan and Holmström, 2003).

APPLICATION OF THE MODEL IN PT. DI PROBLEM

PT. DI has a contract to make wing components for Airbus A380 until 2010. All A380's wing components are packed in 34 types of boxes and are sent by plane, every two weeks. This paper uses the results from research performed by Kustiawan (2007). In his research, Kustiawan (2007) stated that all boxes can be loaded in one container, L5G1 type, only.

Based on the condition in the company the model will help them to minimize the gap of the coordinates of the cargos' and container's center point of mass. However, there is additional problem faced by PT. DI in the placement of boxes into the container, the position of boxes cannot be reversed. The top side of the box must remain face up and the bottom must remain face down. Therefore, the model should be developed to facilitate the problems.

Chen, Lee and Shen (1995) on Kocjan and Holmström (2003) propose modeling rotation of boxes by maintaining additional binary variables indicating that length, width and height of a box is parallel to one of the X (length of the container), Y (width of the container) and Z (height of the container) axis. Such approaches require three binary variables for each side of the box i, where:

- $l_{x_i}, l_{y_i}, l_{z_i}$ is 1 if the width of box *i* is parallel to X, Y or Z, respectively.
- $w_{x_i}, w_{y_i}, w_{z_i}$ is 1 if the width of box *i* is parallel to X, Y or Z, respectively.
- $h_{x_i}, h_{y_i}, h_{z_i}$ is 1 if the width of box *i* is parallel to X, Y or Z, respectively.

The orientation of container and box on axis x, y, and z used in this paper can be seen in the figure 3, which further clarified in the figure 4.

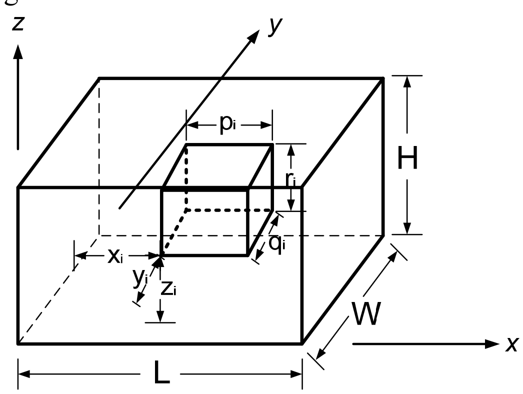


Figure 3 The position of container and box i in xyz coordinates

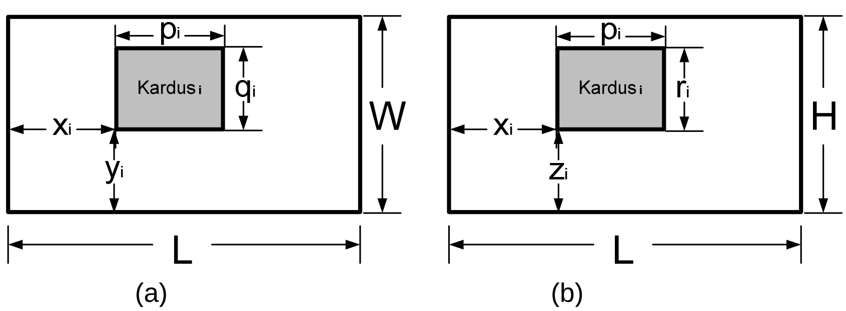


Figure 4 The position of container and box *i* on the coordinate plane xy (a) and on the coordinate plane xz (b)

Since the orientation problem in the company only require that the length or width of boxes parallel with the container's length (x axis) or width (y axis), then the model need a binary variable to define whether the length or the width of the box i is parallel with the length of the container (x axis), change in the objective function, and several changes in the constraints . The additional variable and the changes are following:

a. Decision variable

   o_i binary variable with value of 1 if length of box i is parallel to length of the container (axis x).

b. Objective function

$$minimize \left| \frac{\sum_{i=1}^{N} m_i \cdot (x_i + (\frac{p_i.o_i + l_i.(1-o_i)}{2}))}{\sum_{i=1}^{N} m_i} - \frac{P}{2} \right| + \left| \frac{\sum_{i=1}^{N} m_i \cdot (y_i + (\frac{l_i.o_i + p_i.(1-o_i)}{2}))}{\sum_{i=1}^{N} m_i} - \frac{L}{2} \right|$$

$$+ \left(\frac{\sum_{i=1}^{N} m_i \cdot (z_i + (\frac{t_i}{2}))}{\sum_{i=1}^{N} m_i} \right) \qquad (24)$$

c. Constraints

$$x_i + p_i \cdot o_i + q_i \cdot (1-o_i) \le x_k + (1-a_{ik}) \cdot M \quad \text{for } k = 1,\dots, N,\ i = 1,\dots, k\text{-}1 \qquad (25)$$

$$x_k + p_k \cdot o_k + q_k \cdot (1-o_k) \le x_i + (1-b_{ik}) \cdot M \quad \text{for } k = 1,\dots, N,\ i = 1,\dots, k\text{-}1 \qquad (26)$$

$$y_i + p_i \cdot (1-o_i) + q_i \cdot o_i \le y_k + (1-c_{ik}) \cdot M \quad \text{for } k = 1,\dots, N,\ i = 1,\dots, k\text{-}1 \qquad (27)$$

$$y_k + q_k \cdot (1-o_k) + q_k \cdot o_k \le y_i + (1-d_{ik}) \cdot M \quad \text{for } k = 1,\dots, N,\ i = 1,\dots, k\text{-}1 \qquad (28)$$

$$x_i + p_i \cdot o_i + q_i \cdot (1-o_i) \le L_j \quad \text{for } k = 1,\dots, N,\ i = 1,\dots, k\text{-}1 \qquad (29)$$

$$y_i + q_i \cdot (1-o_i) + q_i \cdot o_i \le W_j \quad \text{for } k = 1,\dots, N,\ i = 1,\dots, k\text{-}1 \qquad (30)$$

   The constraints (25) – (28) ensure that no two boxes placed in the container overlap with each other and the length or width of boxes are parallel with the axis x or y, while the height of boxes must be parallel with the axis z. Constraints (29) – (30) guarantee that boxes are placed within the physical size of the container and consider the orientation limitation.

Implementation has been conducted on a personal computer with a Intel® Pentium® 4 CPU 2,40 GHz and 512 MB by using Extended LINGO Release 8.0 (June 9th, 2004). The results obtained are as follows:

1. Decision variables associated with the boxes i (x_i, y_i, z_i, and o_i)

   The values of the decision variables can be seen in Table 1below.

Table 1 The values of the decisions variable associated with the boxes

Box Type	i	Decision Variables			
		x_i	y_i	z_i	o_i
1	1	5276	127	0	1
3	2	2436	32	1540	0
5	3	676	642	0	1
7	4	11056	307	0	1
9	5	2776	487	0	1
11	6	1216	672	1540	1
13	7	8256	182	0	1
15	8	11056	437	1240	1
16	9	11056	1207	1240	1

17	10	10616	72	1540	1
18	11	10616	72	1920	1
19	12	9396	942	1540	1
20	13	9396	942	2080	1
21	14	4556	127	0	1
22	15	4556	127	80	1
23	16	1306	1622	1200	1
24	17	1306	852	1200	1
25	18	9396	122	1540	1
26	19	9396	122	1720	1
27	20	11586	122	1540	1
28	21	11586	122	1710	1
29	22	1606	1672	1390	1
30	23	1606	952	1390	1
31	24	1256	22	0	1
32	25	1256	22	180	1
33	26	4136	457	1540	1
34	27	4136	457	2070	1

2. The coordinates of center point of mass of cargo (x_{pmk}, y_{pmk}, z_{pmk}) in units of millimeters = (4312, 1084, 878)
3. The difference of cargo's coordinates center point of mass with container's center coordinates center point of in container, in units of millimeters:
   - Difference in component x = | 4312 mm - 6766 mm | = 2454 mm.
   - Difference in component y = | 1084 mm - 1207 mm | = 123 mm.
   - Difference in component z = 878 mm - 0 = 878 mm.

The arrangement and position of each box in the container can be seen in figure 5. The visualization in figure 5 uses isometric view.

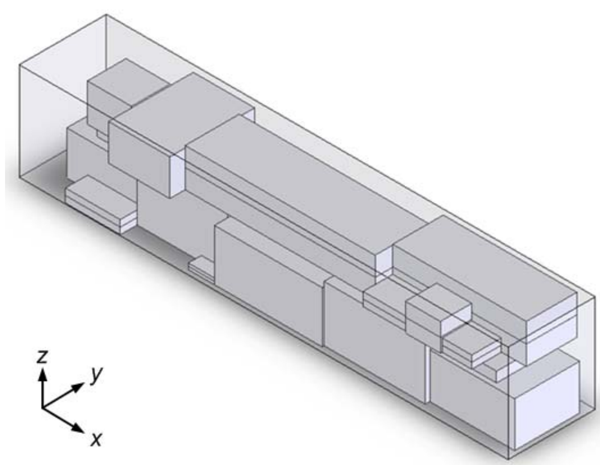
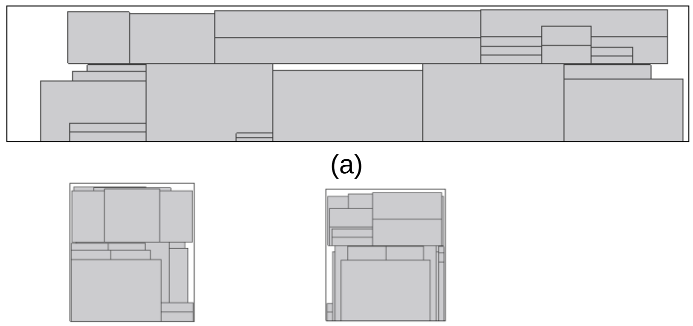


Figure 5 Visualization of the arrangement of 27 boxes of A380 in container L5G1 with isometric view

The visualization of the results is also shown in front, right, and left view in figure 6.



(a)

(b) (c)

Figure 6 Visualization of the arrangement of 27 boxes of A380 in container L5G1 with (a) front view, (b) left view, and (c) right view

The comparison of the center point of mass based on the current condition and the result of this research can be seen on table 2.

Table 2 The comparison of the center point of mass based on the current condition in PT DI and the result of this research in millimeters

Coordinate	Center point of mass of container L5G1	Center point of mass based on the current condition	Center point of mass based on the result of this research
x	6766	4060	4312
y	1207	1583	1084
z	0	907.5	878

The difference between the center point of mass of the container and the cargos is the performance criteria of this research. This research tries to minimize the gap between these points. The gap between these points from the current condition and the result of this paper is shown in table 3 below.

Table 3 The gap comparison of the center point of mass of the container and the cargos based on the current condition in PT DI and the result of this research in millimeters

Coordinate	Current condition	This Research
x	2706	2454
y	376	123
z	907.5	878

Based on the calculation result of the gap between center point of mass of the container and the cargos on the table 3, it is clearly defined that this research can give closer gap of these points. Therefore, this paper gives better cargos arrangement that give better weight distribution than current condition.

CONCLUSION

The problem described in this paper is a problem of allocating a number of boxes of fixed dimension into single container. This paper gives a description of the problem in context of analytical models for packing known from the research literature. The model can consider dimension of all boxes and containers and can find an arrangement of cargos in single container considering weight distribution through center point of mass approach. The arrangement can be found by minimizing the gap of the coordinates of the cargos' and container's center point of mass. Moreover, this paper gives a case example of the application of the model in PT. DI, an airplane manufacturing company in Indonesia. There are several changes in the model to illustrate the company's problem well and to give best solution. Further research could be developed into a form of linear programming, so the computation time required in finding the optimum solution can be reduced. Moreover, it is necessary to build an application to solve CLP in form of Decision Support System (DSS), which can be used repeatedly and consistently, by integrating the input, models, and output user interface, thus can help the decision maker to make decisions more quickly and easily.

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